

Net-baryon-, net-proton-, and net-charged particle kurtosis in heavy ion collisions within a relativistic transport approach

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We explore the potential of net-baryon, net-proton and net-charge kurtosis measurements to investigate the properties of hot and dense matter created in relativistic heavy ion collisions. Contrary to calculations in a grand canonical ensemble we explicitly take into account exact electric and baryon charge conservation on an event-by-event basis. This drastically limits the width of baryon fluctuations. A simple model to account for this is to assume Poisson distributions with a sharp cut-off at the tails. We present baseline predictions of the energy dependence of the net-baryon, net-proton and net-charge kurtosis for central ($b \leq 2.75$ fm) Pb+Pb/Au+Au collisions from $E_{lab} = 2A$ GeV to $\sqrt{s_{NN}} = 200$ GeV from the UrQMD model. While the net-charge kurtosis is compatible with values around zero, the net-baryon number decreases to large negative values with decreasing beam energy. The net-proton kurtosis becomes slightly negative only for low $\sqrt{s_{NN}}$.

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A major aim of today's heavy ion programs at RHIC, CERN and GSI/FAIR is to explore the properties of extremely hot and dense matter. Over the last decade event-by-event fluctuations of observables have been proposed as a novel tool to in heavy ion collisions provide information about the properties of the created matter [1]. Proper observables based on these fluctuations should signal the onset of deconfinement [2, 3, 4], reflecting the change in the degrees of freedom, or the critical point, showing a non-monotonous behaviour with the beam energy [5, 6]. Most fluctuation measures proposed are related to quadratic moments of the event-by-event observables. Among them are charged particle ratio fluctuations, charge transfer fluctuations, baryon number-strangeness correlations and charge-strangeness correlations. Higher moments and susceptibilities, however, become interesting as they are more sensitive to critical phenomena [7] and experimentally accessible. Recent lattice calculations by the RBC-Bielefeld lattice QCD Collaboration [8] reveal that the ratios of higher susceptibilities in net-baryon, net-charge and net-strangeness number are around unity in the low temperature phase, increase towards a maximum at the deconfinement temperature and drop to values of 0 to 0.5 in the deconfined phase. These studies are limited to zero baryo-chemical potential, conditions which can be met at RHIC or LHC.

In this letter we investigate the potential of the net-baryon, net-proton and net-charge kurtosis under experimental conditions and constraints explicitly taking into account electric and baryon charge conservation. Moreover, we provide baseline predictions for the energy dependence of the kurtosis of these quantities. Here the net-charge and the net-proton kurtosis are of special experimental interest. Since the isospin susceptibility re-

mains finite at the critical point critical fluctuations in net-baryon number are reflected in the fluctuations of the net-proton number [9]. Critical phenomena leading to a non-monotonous behaviour on top of this baseline would be a signal for fluctuations due to a phase transition or a critical point in upcoming energy scan experiments at RHIC and the future Facility for Antiproton and Ion Research (FAIR) at GSI. The numerical simulations are done within the microscopic transport model UrQMD in its latest version 2.3 [10, 11, 12]. For previous event-by-event studies within the same model, the reader is referred to [13, 14, 15].

The suggested observables are the quadratic and quartic susceptibilities of baryons (B), protons (P) and charged particles (Q):

$$\chi_2 = \frac{1}{VT^3} \langle \delta N^2 \rangle \quad (1)$$

$$\chi_4 = \frac{1}{VT^3} (\langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2) \quad (2)$$

where $\langle \delta N^4 \rangle = \langle (N - \bar{N})^4 \rangle$ and $\langle \delta N^2 \rangle = \langle (N - \bar{N})^2 \rangle$ are the fourth and second central moments of the respective distribution. The ratio of the quartic to the quadratic susceptibilities represents the kurtosis $K(\delta N)$ times the variance of the distribution, a well known statistical quantity:

$$\frac{\chi_4}{\chi_2} = \frac{\langle \delta N^4 \rangle}{\langle \delta N^2 \rangle} - 3\langle \delta N^2 \rangle = K(\delta N) \langle \delta N^2 \rangle \equiv K^{\text{eff}}. \quad (3)$$

Analysing the effective kurtosis instead of the kurtosis itself has the advantage that it eliminates the $1/N$ behaviour as expected from the central limit theorem and removes the explicit dependence on centrality. The kurtosis of a Gauss distribution is zero, $K_{\text{Gauss}} = 0$. In

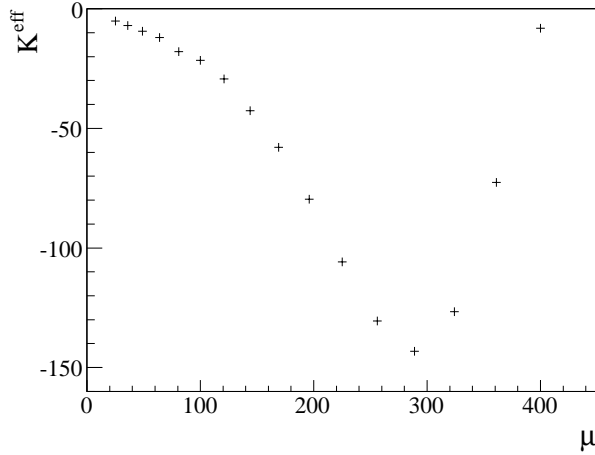


FIG. 1: Effective kurtosis as from a cut Poisson distribution. The cut parameter evolves with the expectation value μ like in equation (5). For increasing μ the effective kurtosis first decreases until the the cut is predominantly limited by $N_{\text{tot}}^{B-\bar{B}}$.

comparison a distribution with $K > 0$ (leptokurtic) has more weight in the peak and tail regions, while a distribution with $K < 0$ (platykurtic) has more weight in the flanks. Thus, platykurtic distributions are generally broader but have a lower peak and more narrow tails.

Let us first explore what to expect in a transport model study of heavy ion collisions. While the lattice QCD calculations are performed in the grand canonical ensemble, transport approaches reflect the micro canonical nature of the underlying single scattering events even in the thermodynamic limit. The choice of the thermodynamic ensemble, however, affects the fluctuations, e.g. discussed in [16]. In a grand canonical ensemble fluctuations of conserved charges are Poisson (or Gauss) distributed and the net charges are conserved only on average, while in heavy ion collisions net-baryon number is conserved exactly in each event. The fluctuations of net-baryon number in a subsystem are of the order of the root of the average net-baryon number in this subsystem. To a first approximation the exact conservation of net-baryon number is manifested in a cut-off at the tails of the net-baryon distributions. What does this imply for the kurtosis analysis of heavy ion data in comparison to lattice QCD calculations? To provide a simple example we introduce a cut in the Poisson distribution for the net-baryon number

$$P_{\mu}(N, C) = \mathcal{N}(\mu, C) e^{-\mu} \frac{\mu^N}{N!} \quad (4)$$

on an interval $[\mu - C, \mu + C]$ where μ is the expectation value of the original Poisson distribution, $C > 0$ indicates the position of the tail cut and $\mathcal{N}(\mu, C)$ is a factor to normalize the modified distribution to unity. To model a (semi-)realistic case we assume that fluctuations in net-baryon number N are restricted even in a

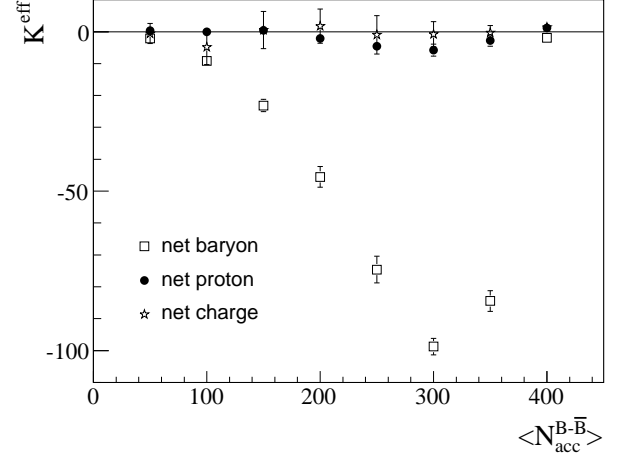


FIG. 2: Effective kurtosis of the net-charge, net-proton and net-baryon number distributions as a function of the width of the rapidity window as calculated from UrQMD for central Pb+Pb reactions at 158A GeV.

comparably small rapidity window. In contrast to statistical fluctuations, an increase of the rapidity window (and therefore of the average net-baryon number in the window) does not lead to stronger fluctuations. On the contrary, when the upper limit of the total net-baryon number N_{tot} is approached the distribution changes to a δ -function ($K_{\delta}^{\text{eff}} = 0$) due to baryon number conservation. To capture this feature we thus assume a simple relation for the cut parameter

$$C = \alpha \sqrt{\mu} \left(1 - \left(\frac{\mu}{N_{\text{tot}}} \right)^2 \right). \quad (5)$$

This means, fluctuations are statistical for a small subsystem and vanish in the limit of full phase space coverage.

Since the Poisson distribution is a discrete distribution on integers, the lower integer number of $\mu - C$ and $\mu + C$ is taken for explicit calculations.

The resulting shape of the effective kurtosis is plotted in Fig. 1 with $\alpha = 3$ and $N_{\text{tot}} = 416$. It can be seen that for this choice the effective kurtosis of the cut Poisson distribution is always negative and decreases with increasing μ until the cut affects the flanks of the distribution and weight is predominantly shifted to the peak leading to an increase of the effective kurtosis. The same qualitative picture can be drawn for a Gamma distribution and the kurtosis of a cut Gauss distribution is always negative. The exact numerical values of the kurtosis crucially depend on the cut and finer details of the respective distribution.

To study the quantitative behaviour let us now turn to the UrQMD investigation of the effective kurtosis. Here we vary the average net-baryon number directly by changing the size of the rapidity window at fixed energies. In Fig. 2, we show the effective kurtosis of the

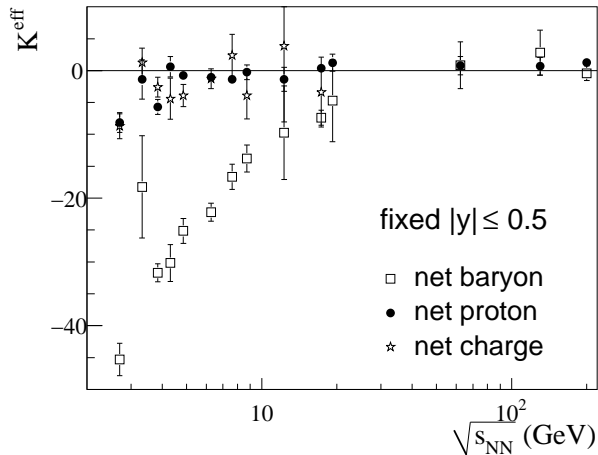


FIG. 3: Effective kurtosis for the net-charge, net-proton and net-baryon number distributions at midrapidity ($|y| < 0.5$) as calculated from UrQMD at various beam energies for central Pb+Pb/Au+Au reactions. The net-charge number at higher energies fluctuates severely due to limited statistics and is removed from this plot for clarity.

net-charge, net-baryon and net-proton number distributions as a function of the rapidity window size in central Pb+Pb reactions at 158A GeV. The latter is expressed by the mean net-baryon number in the acceptance, $\langle N_{\text{acc}}^{B-\bar{B}} \rangle$. The acceptance windows are symmetric around midrapidity. Broadening the rapidity window and thus going to larger mean net-baryon number leads to a decrease of the effective kurtosis until the size comes close to a 4π acceptance (in which the net-baryon number is strictly conserved, $N_{\text{tot}}^{B-\bar{B}} = 416$), and the kurtosis shows the expected steep increase. It is remarkable how well the shape reflects the assumption of a cut Poisson distribution.

The effect of baryon number conservation is also seen in the energy dependence of the effective kurtosis. When going to lower energies, the constant rapidity cut implies that a larger fraction of the total (conserved) baryon charge is observed. Fig. 3 shows the effective kurtosis of the net-charge, net-proton and net-baryon number distributions at midrapidity ($|y| < 0.5$) at a range of beam energies from AGS to RHIC for central Au+Au/Pb+Pb reactions. Here one clearly observes the negative values for the net-baryon number fluctuations decreasing even further for lower energies. At RHIC energies the net-proton and the net-baryon number effective kurtosis is compatible with values of the order 1 and therefore in accordance with the latest data from STAR [17].

If the same fraction of net baryons is chosen (e.g. an average number of 150 net baryons is found in the acceptance $|y| < 0.6$ at 40A GeV and $|y| < 0.85$ at 158A GeV, respectively), the resulting values for the kurtosis are comparable for all investigated energies (see Fig. 4).

Since protons contribute a large amount to the baryon number we expect a similar but less pronounced effect

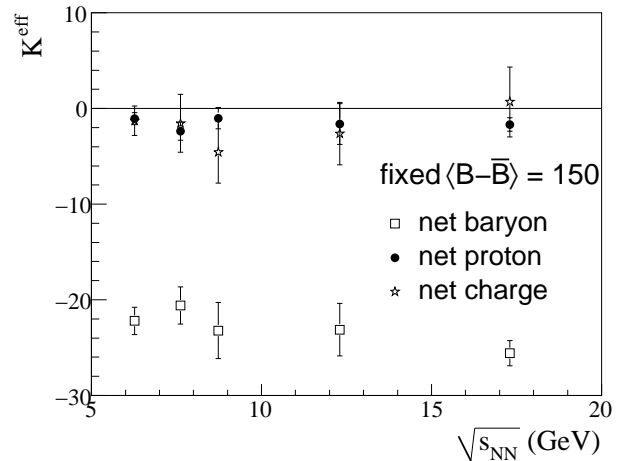


FIG. 4: Effective kurtosis for net-charge, net-proton and net-baryon number distributions in acceptance windows around midrapidity covering an average net-baryon number of 150 at each energy, as calculated from UrQMD at various beam energies for central Pb+Pb reactions.

in their distributions. Only for lower AGS energies in Fig. 3 the effective net-proton kurtosis becomes slightly negative resembling the constraint on net-baryon number. Also in Fig. 2 the effective net-proton kurtosis slightly follows the trend of the effective net-baryon kurtosis. Net charge is also a conserved quantity in a 4π acceptance but the fluctuations are less affected by this constraint. This is due to the large number of pions in a fixed rapidity window that do not contribute to the average net charge but crucially determine the variance. Therefore net-charge fluctuations are less restricted by the constraint on the average net charge, which is determined by the protons. In our picture this is reflected in a less restrictive cut of the distribution. Even close to 4π charge fluctuations are still not completely suppressed. Therefore, the effective net-charge kurtosis is essentially not influenced by the conservation constraint, neither in Fig. 2 nor (within the large error bars) in Fig. 3.

In RQMD [18] and HIJING [19] calculations at these energies we have also found negative values for the effective kurtosis of the same order as are observed in UrQMD calculations.

We have studied the energy dependence of the effective kurtosis of net-baryon, net-proton and net-charge number within a transport model of heavy ion collisions that conserves charge and baryon number explicitly in each event. The net-baryon kurtosis was found to be negative for all energies. This observation, which is surprising in view of the lattice results given in [8] was explained by the exact baryon number conservation in heavy ion collisions. A variation of the rapidity window supported this interpretation and also a simple cut in the tails of a Poisson distribution can reflect the constraint on baryon number fluctuations qualitatively. A negative kurtosis shows that

moderate fluctuations are enhanced while the effect of large fluctuations is suppressed. A realistic treatment of heavy ion collisions at low $\sqrt{s_{NN}}$ and, thus, large baryochemical potential yields drastic differences as compared to the lattice QCD calculations with respect to the net-baryon kurtosis. This is especially worrisome, because the onset of deconfinement is most probably located in this energy region. In our investigation, the effect of baryon number conservation alone is larger than the difference stemming from a (potential) change of the underlying degrees of freedom as suggested by lattice QCD. This is, however, different for the net-proton kurtosis and its potential to be a signal of the critical point and the first order phase transition. Down to low SPS energies the background seen in the effective net-proton kurtosis is only of the order ± 1 and thus small compared to the expected signal from critical fluctuations [7]. However, there are more possible effects in the experimental setup that might influence the kurtosis and support the

trend to negative values, such as the selection of centrality classes and the mixing of distributions for single events. These influences will be subject to further studies. The upcoming experiments of energy scan programs at RHIC and at FAIR will show if non-monotonous deviations from this baseline can be extracted signalling the phase transition or the critical point of QCD.

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